

# Full Manuscript Title: Concise, Specific, and Informative

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**Received** Editorial Office Use

**Accepted** Editorial Office Use

**Published** Editorial Office Use

## ABSTRACT

Write a concise structured abstract of 150–300 words. Clearly state the problem addressed, the objective of the study, the methodology employed, the key results obtained, and the main conclusions drawn. Avoid citations, undefined abbreviations, and non-standard symbols. The abstract must be self-contained and readable independently of the full manuscript. For empirical, numerical, or experimental studies, include at least one quantitative result when possible.

**Keywords:** *keyword one; keyword two; fractional calculus; physics-informed neural networks; keyword five*

## 1 Introduction

Provide background and motivation for the study. Clearly identify the research gap, state the specific objectives, and outline the structure of the paper. Cite relevant prior work using IEEE style, for example [1].

Subsequent paragraphs should provide deeper context. Each paragraph should make one clear point and should flow logically into the next. The final paragraph of the introduction should state the main contribution and briefly describe what each remaining section covers.

## 2 Mathematical Formulation

### 2.1 Governing Equations

The general conservation equation for a transported scalar  $\phi$  in a domain  $\Omega$  is

$$\frac{\partial \phi}{\partial t} + \nabla \cdot (\mathbf{u} \phi) = \nabla \cdot (\Gamma \nabla \phi) + S_{\phi}, \quad (1)$$

where  $\phi$  is the conserved quantity,  $\mathbf{u}$  is the velocity field,  $\Gamma$  is the diffusion coefficient, and  $S_{\phi}$  is the source term. The fractional-order extension of (1), using the Caputo derivative of order  $\alpha \in (0, 1]$ , is

$${}^0D_t^{\alpha} \phi + \nabla \cdot (\mathbf{u} \phi) = \nabla \cdot (\Gamma \nabla \phi) + S_{\phi}. \quad (2)$$

### 2.2 Boundary and Initial Conditions

Dirichlet conditions on  $\partial\Omega_1$  are written as

$$\phi = \phi_0, \quad \text{on } \partial\Omega_1, \quad t > 0. \quad (3)$$

Neumann conditions on  $\partial\Omega_2$  are written as

$$\Gamma \nabla \phi \cdot \hat{\mathbf{n}} = q, \quad \text{on } \partial\Omega_2, \quad t > 0. \quad (4)$$

### 3 Numerical Method

#### 3.1 Discretisation

The spatial domain is discretised using [method]. The time-stepping scheme is [scheme] with step  $\Delta t$ . Mesh convergence was verified on successively refined grids.

#### 3.2 Solver and Convergence

The resulting linear system is solved using [solver]. Convergence is declared when the  $\ell_2$  residual falls below  $10^{-6}$ . All computations were performed in [environment] on [hardware].

##### 3.2.1 Stability Analysis

Von Neumann stability analysis yields the following condition:

$$\Delta t \leq \frac{C (\Delta x)^2}{\Gamma}, \quad (5)$$

where  $C$  is a constant depending on the fractional order  $\alpha$ .

### 4 Results

#### 4.1 Verification and Validation

Figure 1 compares the numerical and analytical solutions for the limiting case  $\alpha \rightarrow 1$ . The maximum relative error is below 0.1% across all tested grid resolutions, confirming second-order spatial accuracy.

[ Insert Figure 1 here ]

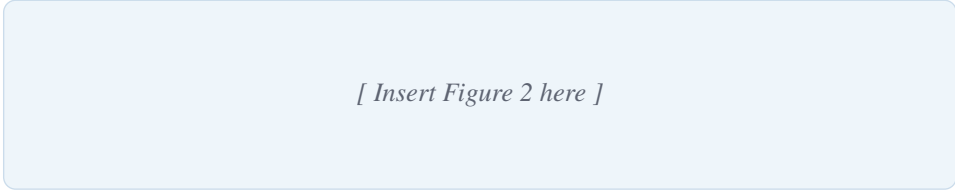
**Figure 1.** Comparison of numerical and analytical solutions for  $\alpha = 0.8, 0.9, \text{ and } 1.0$ . Symbols: numerical; lines: analytical.

#### 4.2 Parametric Study

Table 1 summarises the key results. Figure 2 shows the spatial distribution of  $\phi$  at  $t = 1.0$  for three values of  $\alpha$ .

**Table 1.** Summary of numerical results for selected parameter combinations.

Parameter	$\alpha = 0.70$	$\alpha = 0.85$	$\alpha = 1.00$	Reference
$L_2$ error	$3.2 \times 10^{-3}$	$1.8 \times 10^{-3}$	$0.9 \times 10^{-3}$	—
CPU time (s)	12.4	10.8	9.2	—
Convergence order	1.94	1.97	2.00	2.00
Maximum error	$5.1 \times 10^{-3}$	$2.7 \times 10^{-3}$	$1.4 \times 10^{-3}$	—



**Figure 2.** Spatial distribution of  $\phi$  at  $t = 1.0$  for  $\alpha = 0.70, 0.85,$  and  $1.00$ .

## 5 Discussion

Interpret the results in the context of the research objectives and existing literature. Discuss the physical significance of observed trends, the limitations of the study, and the broader applicability of the findings. Comparison with prior work [2, 3] shows that the proposed approach achieves comparable accuracy with significantly lower computational cost. The fractional-order formulation captures non-Fickian transport behaviour that classical integer-order models cannot reproduce.

## 6 Conclusions

The following conclusions are drawn from this study:

- First conclusion: concise statement of the main finding.
- Second conclusion: methodological contribution or validation outcome.
- Third conclusion: practical implication or future research direction.

### FUNDING

This research was funded by [Funding Agency], grant number [XXXX]. / This research received no external funding.

### AUTHOR CONTRIBUTIONS (CREDIT)

<b>Conceptualization</b>	<i>A.O.</i>	<b>Methodology</b>	<i>A.O., A.T.</i>
<b>Software</b>	<i>A.O.</i>	<b>Validation</b>	<i>A.T.</i>
<b>Formal analysis</b>	<i>A.O.</i>	<b>Investigation</b>	<i>All authors</i>
<b>Writing – original draft</b>	<i>A.O.</i>	<b>Writing – review and editing</b>	<i>All authors</i>
<b>Visualization</b>	<i>A.T.</i>	<b>Supervision</b>	<i>A.O.</i>
<b>Project administration</b>	<i>A.O.</i>	<b>Funding acquisition</b>	<i>A.O.</i>

### DATA AVAILABILITY STATEMENT

Data supporting the findings are available from the corresponding author upon reasonable request. / All data are publicly available at [repository URL, DOI].

### CONFLICT OF INTEREST

The authors declare no conflict of interest.

### ACKNOWLEDGEMENTS

The authors thank [individuals / institutions] for their valuable support and contributions to this work.

### AI Use Declaration

The authors used [tool/service name] to support [specific task, e.g., language editing, formatting, or improving clarity]. The output was critically reviewed, revised where necessary, and verified by the authors. The authors accept full responsibility for the accuracy, integrity, and originality of the final manuscript. If no such tools were used, replace this statement with: No generative AI or AI-assisted technologies were used in preparing this manuscript.

### References

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